X10655   Reg. No. :

Question Paper Code : X 10655

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020/
APRIL/MAY 2021
Second Semester
MA8251 : ENGINEERING MATHEMATICS – II
[Common to all (Except Marine Engineering)]
(Regulations 2017)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. Given that α, β are the eigenvalues of the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$, form the matrix whose eigenvalues are α^2, β^2.

2. If the canonical form in the three variables u, v, w is given by $3v^2 + 15w^2$ corresponding to a quadratic form, then state the nature, index, signature and rank of the quadratic form.

3. Check whether the vector
$\vec{F} = \left(y^2 - z^2 + 3yz - 2x\right)\hat{i} + \left(3xz + 2xy\right)\hat{j} + \left(3xy - 2xz + 2z\right)\hat{k}$
is solenoidal or not.

4. State Green's theorem in a plane.

5. State the polar form of the Cauchy Riemann equations.

6. Find the invariant points of the mapping $w = \dfrac{z - i}{1 - iz}$.

7. State the Taylor series representation of an analytic function f(z) about z = a.

8. State the nature of the singularity of $f(z) = z \cos\left(\dfrac{1}{z}\right)$.

9. Using Laplace transform of derivatives, find the Laplace transform of $\cos^2 t$.

10. Given $L\{f(t)\} = \dfrac{1}{s\left(s+1\right)(s+2)}$, find $\lim\limits_{t \to 0} f(t)$.

PART – B **(5×16=80 Marks)**

11. a) i) The eigenvectors of a real symmetric matrix A corresponding to the eigenvalues 2, 3, 6 are respectively $(1, 0, -1)^T$, $(1, 1, 1)^T$ and $(-1, 2, -1)^T$. Find the matrix A. **(8)**

ii) Show that A satisfies its own characteristic equation and hence find A^8 if $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$. **(8)**

(OR)

b) i) Using Cayley-Hamilton theorem, find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$. **(8)**

ii) Reduce the quadratic form $3x^2 + 2y^2 + 3z^2 - 2xy - 2yz$ into a canonical form using an orthogonal transformation. **(8)**

12. a) i) Find the angle between the normals to the surface $xy = z^2$ at the points $(-2, -2, 2)$ and $(1, 9, -3)$. **(6)**

ii) Verify Stokes' theorem for $\vec{F} = xy\hat{i} - 2yz\hat{j} - zx\hat{k}$ where S is the open surface of the rectangular parallelopiped formed by the planes $x = 0$, $x = 1$, $y = 0$ $y = 2$ and $z = 3$ above the xoy-plane. **(10)**

(OR)

b) i) Find the values of a, b, c so that $\vec{F} = (axy + bz^3)\hat{i} + (3x^2 - cz)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. For these values of a, b, c, find also the scalar potential of $\vec{F}$. **(8)**

ii) Using Gauss' divergence theorem, evaluate $\iint_S \vec{F}\cdot\hat{n}\, dS$ where $\vec{F} = y\hat{i} + x\hat{j} + z^2\hat{k}$ and S is the surface of the cylindrical region bounded by $x^2 + y^2 = a^2$, $z = 0$ and $z = b$. **(8)**

13. a) i) Show that $u = e^x \cos y$ is harmonic. Find the analytic function $w = u + iv = f(z)$ using Milne-Thompson method and hence find the conjugate harmonic function v. **(10)**

ii) Given $w = u + iv = z^3$, verify that the families of curves $u = C_1$ and $v = C_2$ cut orthogonally. **(6)**

(OR)

b) i) If f(z) is an analytic function, then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f'(z)|^2$. **(10)**

ii) Find the image of the triangular region in the z-plane bounded by the lines x = 0, y = 0 and x + y = 1 under the transformation $w = e^{i\pi/4}z$ **(6)**

14. a) i) If $f(a) = \oint_C \frac{3z^2+7z+1}{z-a}\,dz$, C is the circle | z | = 2, then find the values of f(3), f′(1 + i) and f″(1 – i). **(8)**

ii) Using Laurent's series expansion, find the residue of $f(z) = \frac{z^2}{(z-1)(z+2)^2}$ at its simple pole. **(8)**

(OR)

b) Using contour integral, evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)^2(x^2+2x+2)}$. **(16)**

15. a) i) Using Laplace transform, evaluate $\int_0^{\infty}\left(\frac{\cos at - \cos bt}{t}\right)dt$. **(8)**

ii) Using convolution theorem, find $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$. **(8)**

(OR)

b) i) Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} t & 0 \le t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$$ with period 2a. **(8)**

ii) Using Laplace transform, solve $(D^2 + 4D + 13)y = e^{-t}\sin t$ given $y = Dy = 0$ at $t = 0$, $D \equiv \frac{d}{dt}$. **(8)**

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